Short Communication:

Taxation Based on Social Norms: An Axiomatic Approach

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Abstract

We show in this paper how a tax law can be formulated which (i) does not distort economic incentives of individuals and firms, (ii) is based on low information requirements, and (iii) which does not violate important behavioral facts such as fairness in taxation of the subjects. That for, a social norm approach is used which replaces the individuals' utility functions in optimal taxation theory initiated by Vickrey (1945) and Diamond and Mirrless (1971). Based on axioms representing the social norms, we prove that a unique class of tax functionals exists. Furthermore, the tax functionals can be explicitly constructed and possess a concrete integral representation. This class of admissible tax functionals which are compatible with the given axioms allows to determine a tax functional which optimizes an externally given objective function. We also compare the theory to the actual Swiss tax law. In doing so, we find that all but one axiom are violated. More specifically, tax functional induced by the current law is neither monotone, nor continuous, nor additive nor time invariant. Finally, the tax law also violates the economic incentive compatibility axiom.

Keywords and phrases: Taxation, axiomatic tax law, incentive compatibility

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1 Introduction

There exists an extensive literature in economic theory about "optimal taxation"[3]. These theories are based on preferences of individuals, firms and the government. It is assumed in the models initiated by the work of Vickrey and Mirrless that the government's objective is to maximize the sum of all individuals' utility under various restrictions. These models are further characterized by asymmetric information: In the models of Vickrey (1945) and Diamond and Mirrless (1971) for example, individuals know their productivity, but this information is not shared by the government. These preferences finally form optimization programs which lead to the optimal taxation rules. Besides the asymmetric information, the assumption that the government knows the preferences of the whole population is very strong.

The approach proposed in this paper is based on *aggregate preferences*. These preferences which can be called *social norms* or *commonly believed rules of fairness*[4] possess the advantage of replacing the population of individuals by a single individual defined by its social preferences. This considerably diminishes the information problems of the models mentioned above. The drawback of such an aggregation procedure is the lack of explicitly defined preferences for the individuals which drive the form of the optimal taxation rules in the literature. Hence, we have to make sure, that the behavior of the social norm individual is in line with the behavior of the individual on the microeconomic level. Therefore, we postulate two basic principles for the social preferences:

> **Postulate 1:** *The social norm should not implement an incentive not to work.* (This is weaker than to say that it should implement an incentive to work).
>
> **Postulate 2:** *The social norm should allow to discriminate between fair and unfair taxation.*

These principles are formalized in an axiomatic system for the tax functional. Hence, we do not model social preferences per se, but we directly axiomatize properties which a tax functional should possess. These axioms are *minimal* requirements on the tax functional such that we may assume that the population agrees on these principles. The approach is set up as follows: First, we define the theory's objects which are, in our model case, income and capital. Second, we state axioms which describe how the objects matter.

The goal is then in a first step to prove that a tax functional which is compatible with the axioms exists. Indeed, we will deduce from the axioms some necessary conditions on the tax functional from which it will follow (in Theorem 2) that the functional must have a very particular form, namely it has a specific and explicit integral representation which involves a rating function with must satisfy certain conditions (see Lemma 1). Conversely we will see that these conditions are also sufficient for a tax functional. In other words, every functional with a rating function satisfying the mentioned conditions *is* a tax functional compatible with the axioms (see Theorem 4). The possible choices for the rating function include that of a linear function which leads to a linear taxation theory, but our approach also includes nonlinear taxation.

How can the rating function be determined? So far, optimality was not an issue. Up to now, our only concern was whether for given social norms a taxation functional exists and what its properties are. If we compare this with the theory of optimal taxation, at this point the government is free to choose the remaining parameters (i.e. the rating function) in such a way that governmental

[3]See Mirrless (1991) for an excellent survey or Dasgupta and Stiglitz (1972), Diamond and Mirrless (1971), Guesnerie (1977), Hahn (1973), Mussa and Rosen (1978), Myerson (1982), Spence (1977, Vickrey (1945, 1968).

[4]The importance of fairness and reciprocity is discussed in Akerlof and Yellen (1988), Camerer and Thaler (1995), Fehr and Gaechter (1998), Güth (1995), Levine (1993), Roth (1995).

objective is optimized. In view of this interpretation of the rating function, we might call it as well *government policy function*. Contrary to the classical optimal taxation theory, the objective needs not to be the sum of individuals' utility but can be a properly chosen utility function representing the government's needs and goals. In this form, the use of social norms is a resolution of the asymmetric information problems in classical optimal taxation theory. In other words, the social norm approach determines the form of the tax functional and the class of admissible functions for the final optimization of the government. This is an ordinary optimization under symmetric information with the government the only optimizer. The form of the tax functional then makes sure that the aggregated individuals' objective are not violated in this final program.

This standard optimization problem is not carried out. Instead, we shortly compare the taxation functional with the actual taxation law in Switzerland. Surprisingly, the Swiss taxation law violates all but one of the axioms defining the social norms. A detailed analysis of the Swiss taxation law in the light of the social norm approach is given in Hungerbühler and Vanini (2001).

The paper is organized as follows. In Subsection 2.1, we give a model definition of the basic objects of the tax law. In Subsection 2.2, we present some basic and plausible axioms for the tax functional, based on the Postulates 1 and 2. In Section 2.3, we derive properties of the tax functional from the axioms. The main conclusion is the existence and uniqueness of a tax functional and its explicit construction. We also show that the found conditions completely characterize the class of all tax functionals compatible with the axioms. Finally, in Subsection 2.4, the actual Swiss taxation law is compared with the axiomatic approach. In Section 3 we summarize the results.

2 The model

We proceed in the same way as in other axiomatized theories which are based on a three-step-procedure: The first step is to isolate the basic objects of a theory, like "consumption set" in decision theory or "event space" in probability theory. The second step is to formulate in a number of axioms the relations between these objects, e.g., "transitivity of the binary relation". The third step is to build the theory from these axioms by logic deduction, like, e.g., the von Neumann-Morgenstern representation theorem in decision theory.

2.1 Basic objects of the tax law

Traditionally, "income" and "capital" are rated by the state. Further, taxes on consumption goods, energy sources and legal addictive goods exist. In this paper the discussion is restricted to income and capital, however it will become clear that the method works equally well for any other sort of tax raising. We will use "income" and "capital" only as *ad hoc* expressions which are not identical to the expressions used in accounting.

We first consider the notion of capital. This is the wealth $k(t)$ (measured in currency units), which an individual or a firm possesses at time t in form of a bank account or of immovables for example. If there are different categories of capital (e.g., when bank accounts undergo a different tax rate than immovables), then of course one has for each category of capital a separate time dependent variable $k_i(t)$. We adhere to only one capital category, since the theory carries over to the case of diverse categories in a straight forward way.

Income $e(t_0, t_1)$ (measured in currency units) is the integral function of the wage rate $w(t)$ (measured in currency units per time unit), more precisely,

$$e(w, t_0, t_1) = \int_{t_0}^{t_1} w(\tau) d\tau$$

is the realized income[5] in the time interval $[t_0, t_1]$. Although the wage is usually paid in discrete steps (at the end of each month) it is useful to represent $w(t)$ as a density function[6]. Again, if more than one category of income or wage are taken into consideration one will have to assign a separate variable to each category.

Hence, capital and income will be described by the two real valued functions $k(t)$ and $w(t)$, respectively.

The taxes for a time interval $[t_0, t_1]$, $t_1 \geqslant t_0$, are then a real valued functional $\mathscr{T}(k, w, t_0, t_1)$. For natural individuals this functional is typically the sum of the income taxes $\mathscr{I}(w, t_0, t_1)$ and a capital taxation $\mathscr{C}(k, t_0, t_1)$. As admissible functions w and k, we choose the functions in the vector space $L^1_{\mathrm{R}}(\mathbb{R})$, which is the set of Riemann integrable functions, equipped with the L^1-topology.

2.2 Axioms of the tax law

We first state the axioms for the tax functional $\mathscr{T}(k, w, t_0, t_1)$ which should hold true. This is a suggestion by the authors and clearly new axioms can be added or existing axioms replaced by others.

Axiom 1: Positivity. $\mathscr{T} \geqslant 0$.

This first axiom is a plausible demand: The government will not *pay* an individual or firm if, for example wage and capital are both negative (indeed, such a policy would violate Postulate 1 in Section 1). In other words, there is no "negative tax". In fact, this axiom is fulfilled by all tax laws known to us.

Axiom 2: Continuity. *The functional $\mathscr{T}$ should continuously depend on its last variable t_1.*

Intuitively, this axiom excludes that somebody who is only *slightly* longer (e.g. one day) taxable than somebody else pays a *sharp* higher tax. Indeed, jumps in the taxation of this kind would violate Postulate 2.

The other axioms will imply the continuity of $\mathscr{T}$ in the variables w, k and t_0 (see Subsection 2.3). Hence, the continuity property of the tax functional with respect to these variables is not required in Axiom 2.

Axiom 3: Monotonicity. *The functional $\mathscr{T}$ is monotone increasing in the variables k and w, i.e.,*

$$\begin{aligned} k_1 \geqslant k_2 \text{ on } [t_0, t_1] \quad &\Longrightarrow \quad \mathscr{T}(k_1, w, t_0, t_1) \geqslant \mathscr{T}(k_2, w, t_0, t_1) \\ w_1 \geqslant w_2 \text{ on } [t_0, t_1] \quad &\Longrightarrow \quad \mathscr{T}(k, w_1, t_0, t_1) \geqslant \mathscr{T}(k, w_2, t_0, t_1)\ . \end{aligned}$$

[5] We set all discount factors and interest rates equal to unity.

[6] For example, for a month wage earning of h, we set $w(t) \equiv 12h$ for this month if we consider the calendar year as time unit.

Hence, somebody who earns less (or possesses less capital) than somebody else also pays less taxes in absolute values. Also this axiom is inspired and in accordance with Postulate 2.

Monotonicity is a weaker property than *progression* which means that somebody who earns more has to pay a higher *percentage* of his income as taxes. So, progression would be a candidate for an additional axiom (see the discussion on Axiom 7 in Subsection 2.3). We do not need to postulate the monotonicity for the time variables t_1 (increasing) and t_0 (decreasing) since the monotonicity together with the additivity property below imply these properties (see Section 2.3).

Axiom 4: Economic Incentive Compatibility (Fairness). *An individual which earns more (or has more income), should possess still more after the subtraction of the taxes than somebody who earns less (or has less income). In formulae this reads as follows: if k_1, k_2, w_1, w_2 are constant in time, then*

$$\begin{aligned} k_1 \geqslant k_2 \quad &\Longrightarrow \quad k_1 - \mathscr{T}(k_1, w, t_0, t_1) \geqslant k_2 - \mathscr{T}(k_2, w, t_0, t_1) \\ w_1 \geqslant w_2 \quad &\Longrightarrow \quad e(w_1, t_0, t_1) - \mathscr{T}(k, w_1, t_0, t_1) \geqslant e(w_2, t_0, t_1) - \mathscr{T}(k, w_2, t_0, t_1). \end{aligned}$$

This axiom is crucial from a behavioral point of view. It is evident that a violation of Axiom 4 will be valued by individuals as unfair treatment by the state, and would therefore violate Postulate 2. Due to the importance fairness plays (see footnote 4) this axiom should hold true in any tax law. Axiom 4 does *not* imply that the individual has an incentive to work, but it is in accordance with Postulate 1. It assures, that given such an incentive on the microeconomic level, it is maintained on the aggregated level in the taxation procedure. The next axiom states that taxation has to be time invariant if circumstances are not changing. More precisely,

Axiom 5: Time Invariance. *The taxes for equal time intervals calculated at different point of times are equal given circumstances unchanged. Hence,*

$$\mathscr{T}(k(t), v(t), t_0, t_1) = \mathscr{T}(k(t-T), v(t-T), t_0 + T, t_1 + T)$$

has to hold for arbitrary T.

Again, we can justify this axiom by Postulate 2. Most tax laws try to satisfy this axiom at least partially. Consider for example a worker which only works two months of a year. Then income has to be transformed at an annual rate and then the taxes are calculated according to the corresponding tax rate pro rata temporis for the two months. Finally, we have:

Axiom 6: Additivity. *The taxes of different tax periods add up: We have for $t_0 \leqslant t \leqslant t_1$:*

$$\mathscr{T}(k, w, t_0, t_1) = \mathscr{T}(k, w, t_0, t) + \mathscr{T}(k, w, t, t_1).$$

Given the original six axioms, it is not a priori clear, whether a nontrivial (i.e. except for $\mathscr{T} \equiv 0$) tax functional $\mathscr{T}$ satisfying all axioms exists. Further, if it exists, how does it look like? We answer these questions together with some conclusions from the axioms in the next section.

Thus, the general program is (i) to prove that a (nontrivial) tax functional satisfying the axioms exists, (ii) to construct such a functional and (iii) to describe the class of all possible functionals compatible with the axioms. In the next Section, we give a complete answer to (i)–(iii), given the 6 axioms above.

The existing list of axioms is by no means exhaustive, only the most important ones have been considered. Progression might be another possible axiom. We add this axiom at the end of next section and compare this enlarged model with the original one.

2.3 Conclusions from the axioms

A first simple conclusion from the axioms is monotonicity of the tax functional in the time variables. In fact, for $t_0 \leqslant t_1 \leqslant t_2$, it follows from the additivity and the positivity that

$$\mathscr{T}(k, w, t_0, t_2) = \mathscr{T}(k, w, t_0, t_1) + \mathscr{T}(k, w, t_1, t_2) \geqslant \mathscr{T}(k, w, t_0, t_1).$$

To simplify the forthcoming presentation, we consider the traditional case where the tax $\mathscr{T}$ is the sum of income tax $\mathscr{I}$ and capital tax $\mathscr{C}$, i.e. $\mathscr{T}(w, k, t_0, t_1) = \mathscr{I}(w, t_0, t_1) + \mathscr{C}(k, t_0, t_1)$. We only discuss the income tax $\mathscr{I}$. If we adopt the **normalization** (which might as well be a further axiom)

$$\mathscr{C}(0, t_0, t_1) = 0$$

then all axioms stated for $\mathscr{T}$ hold separately for $\mathscr{I}$, too. This follows by setting $k \equiv 0$ everywhere in the axioms. Also, under the corresponding normalization $\mathscr{I}(0, t_0, t_1) = 0$, the results we obtain below for $\mathscr{I}$ hold accordingly for $\mathscr{C}$.

Let us suppose now, that the taxes are determined for a constant wage $w(t) \equiv w$ in a unit time interval

$$\mathscr{I}(w, t, t+1) =: f(w). \tag{1}$$

In particular, the invariance axiom assures that the **rating function** $f : \mathbb{R} \to \mathbb{R}$ does not depend on t and is hence well defined. To be closer to traditional approaches, we could write $f(w) = g(w)w$, where g represents the tax rate. Progression (see Axiom 7 below) would then state that g is a monotone increasing function.

What properties does the rating function $f(w) = \mathscr{I}(w, t, t+1)$ possess? First, from the positivity axiom, we deduce that f is a non-negative function

$$0 \leqslant \mathscr{I}(w, t, t+1) = f(w).$$

Applying the monotonicity axiom, we get for $w_1 \leqslant w_2$

$$f(w_1) = \mathscr{I}(w_1, t, t+1) \leqslant \mathscr{I}(w_2, t, t+1) = f(w_2),$$

i.e. the rating function f is itself monotone increasing. What follows from the fairness axiom? For $w_1 \geqslant w_2$ follows

$$w_1 - f(w_1) = \int_t^{t+1} (w_1 - f(w_1))d\tau \geqslant \int_t^{t+1} (w_2 - f(w_2))d\tau = w_2 - f(w_2).$$

This implies, that f is Lipschitz-continuous with Lipschitz-constant 1. Summarizing we may state:

Lemma 1 *The axioms imply for the rating function $f(w) := \mathscr{I}(w, t, t+1)$, w constant, that*

(a) $f \geqslant 0$

(b) $w_1 \leqslant w_2 \implies 0 \leqslant f(w_2) - f(w_1) \leqslant w_2 - w_1$.

It follows from Rademacher's Theorem that f is almost everywhere differentiable, and the pointwise derivative f' (which is defined almost everywhere) agrees with the weak derivative of f. Thus, the property (b) can equivalently be expressed by *(b') f is weakly differentiable and $0 \leqslant f' \leqslant 1$.*

The goal is now to represent $\mathscr{I}$ in the form

$$\mathscr{I}(w,t,t+1) = \int_t^{t+1} f(w(\tau))d\tau, \tag{2}$$

which is obviously true for constant w. The additivity and the invariance axiom imply for any rational number $q = \frac{m}{n}$ (and w still constant)

$$\begin{aligned} \mathscr{I}(w,t,t+q) &= \frac{1}{n}\sum_{i=0}^{n-1}\mathscr{I}(w,t+iq,t+(i+1)q) = \frac{1}{n}\mathscr{I}(w,t,t+\underbrace{nq}_{=m}) \\ &= \frac{1}{n}\sum_{i=0}^{m-1}\mathscr{I}(w,t+i,t+i+1) = \frac{m}{n}f(w) = \int_t^{t+q} f(w(\tau))d\tau. \end{aligned}$$

Using the continuity property stated in axiom 2, we can extend this formula to arbitrary time intervals of length $x \in \mathbb{R}$ as follows

$$\begin{aligned} \mathscr{I}(w,t,t+x) &= \lim_{\mathbb{Q}\ni q\to x}\mathscr{I}(w,t,t+q) = \lim_{\mathbb{Q}\ni q\to x}\int_t^{t+q} f(w(\tau))d\tau \\ &= \int_t^{t+x} f(w(\tau))d\tau. \end{aligned} \tag{3}$$

Again, additivity and invariance imply that the formula (3) holds for step functions, i.e. functions of the form $w(\tau) = \sum_{i=0}^{n-1} w_i \chi_{[t_i,t_{i+1}[}(\tau)$[7]. Indeed, we have

$$\mathscr{I}(w,t_0,t_n) = \sum_{i=0}^{n-1}\mathscr{I}(w_i,t_i,t_{i+1}) = \sum_{i=0}^{n-1}\int_{t_i}^{t_{i+1}} f(w_i)d\tau = \int_{t_0}^{t_n} f(w(\tau))d\tau.$$

Finally, we prove the formula for arbitrary wage functions $w \in L^1_{\mathrm{R}}$. For this purpose, we pick two bounded sequences u_n and w_n of step functions on the interval $I = [t_0, t_1]$, such that $u_n \leqslant v \leqslant w_n$ holds on I and such that $\int_I (w_n - u_n)d\tau \to 0$ for $n \to \infty$[8]. Picking an adequate subsequence (still indexed by n), we can assume that u_n and w_n converge almost everywhere on I to w. The Lipschitz-continuity of f implies then that $f(u_n) \to f(w)$ for $n \to \infty$ almost everywhere on I and $f(w_n) \to f(w)$ for $n \to \infty$ almost everywhere on I. Since both sequences are also uniformly bounded on I, the theorem of Lebesgue implies that both sides of the monotonicity inequality

$$\int_{t_0}^{t_1} f(u_n(\tau))d\tau \leqslant \mathscr{I}(w,t_0,t_1) \leqslant \int_{t_0}^{t_1} f(w_n(\tau))d\tau$$

converge to the value $\int_{t_0}^{t_1} f(w(\tau))d\tau$. We note, that this is the only place, where we used the monotonicity axiom for a non-constant wage.

Summarizing, we proved the following theorem:

Theorem 2 *Let $f(w) := \mathscr{I}(w,t,t+1)$ for w constant. Then, if the axioms 1 to 6 are valid for $\mathscr{I}$, we have the following representation formula for the tax functional: For all $w \in L^1_{\mathrm{R}}$ there holds*

$$\mathscr{I}(w,t_0,t_1) = \int_{t_0}^{t_1} f(w(\tau))d\tau. \tag{4}$$

[7] χ_I is the indicator function for the interval I, and $t_0 \leqslant t_1 \leqslant \ldots \leqslant t_n$.
[8] Such sequences exist due to the definition of the Riemann integrable functions.

As a corollary follows the continuity of the tax functional with respect to the variable w:

Corollary 3 *If $w_n \to v$ in L^1_{R} for $n \to \infty$, then $\mathscr{I}(w_n, t_0, t_1) \to \mathscr{I}(w, t_0, t_1)$.*

The proof follows immediately from the representation formula (4) and the properties of the function f stated in Lemma 1. q.e.d.

How rich is the class of tax functionals? So far, the only necessary conditions for the function f are stated in Lemma 1. Indeed, these conditions turn out to be sufficient too. We have

Theorem 4 *If f is a functions satisfying the conditions (a) and (b) (or (b')) of Lemma 1, then (4) defines a tax functional, which satisfies all axioms 1 to 6 stated in Section 2.2. Moreover, f agrees with the rating function of the such defined tax functional.*

The proof of this claim is a straightforward verification of the axioms. q.e.d.

Typically, additional axioms make the class of possible functionals smaller by generating new conditions for the admissible functions f (see, e.g., Lemma 5 below). Another way to explicitly specify the function f is to optimize an objective function h of f (under the given side conditions which f has to satisfy) with respect to some given benchmark function.

Let us come back to the notion of progression, which was already mentioned several times. The corresponding axiom would, e.g., look as follows

> **Axiom 7: Progression.** *If $k_1 \leqslant k_2$ and $w_1 \leqslant w_2$ are constant in time, then, for arbitrary $t_0 \leqslant t_1$,*
>
> $$\begin{aligned} \frac{\mathscr{I}(w_1, t_0, t_1)}{w_1} &\leqslant \frac{\mathscr{I}(w_2, t_0, t_1)}{w_2}, \\ \frac{\mathscr{C}(k_1, t_0, t_1)}{k_1} &\leqslant \frac{\mathscr{C}(k_2, t_0, t_1)}{k_2}. \end{aligned}$$

These inequalities reflect the fact, that somebody who earns more or possesses more than somebody else, has to pay a higher percentage of his or her income or capital in form of income or capital tax. In view of the other axioms, we can easily reformulate the progression axiom and state it for the income tax in terms of the rating function $f(w) = \mathscr{I}(w, t, t+1)$. Indeed we have:

Lemma 5 *If all axioms 1 through 7 are valid for $\mathscr{I}$, then the rating function $f(w) = \mathscr{I}(w, t, t+1)$ satisfies (in addition to the conditions (a) and (b) stated in Lemma 1)*

$$w_1 \leqslant w_2 \quad \Longrightarrow \quad \frac{f(w_1)}{w_1} \leqslant \frac{f(w_2)}{w_2}. \tag{5}$$

Conversely, if f satisfies the conditions (a) and (b) of Lemma 1 and (5), then the functional defined by the representation formula (4) satisfies all axioms 1 through 7.

If we write $f(w) = g(w)v$, then (5) is equivalent to say that the tax rate g is a monotone increasing function.

The strength of an axiomatic tax system is its simplicity. In fact, we only need a juristic definition of the wage function w and an explicit choice of the rating function $f : \mathbb{R} \to \mathbb{R}$ (for example a piecewise affine function with parameters which can be adjusted at specific times according to the financial situation of the state, or by optimizing an externally given objective function), satisfying the conditions (a) and (b) (or (b')) in Lemma 1, is needed to obtain a tax functional $\mathscr{I}$ which is fully compatible with the principles stated as axioms.

Summarizing, we have addressed the following three points: There exists a nontrivial tax functional $\mathscr{I}$ satisfying the axioms. We have constructed the functional form (4) of the tax functional $\mathscr{I}$. We have identified the class of all possible rating functions f consistent with the axioms. In particular it is worth noting that the tax functional is *uniquely* determined (and explicitly given by the representation formula in Theorem 2) by the axioms and the rating function $f(w) := \mathscr{I}(w, t, t+1)$ for constant w. In other words, the only freedom (which is still rich enough) in income taxation is the choice of the function f, subject to the conditions (a) and (b) of Lemma 1. We recall that under the normalization $\mathscr{I}(0, t_0, t_1) = 0$ the axioms also hold separately for the capital tax functional $\mathscr{C}$ and hence our results for $\mathscr{I}$ hold accordingly for $\mathscr{C}$. More precisely, we have the representation formula

$$\mathscr{C}(c, t_0, t_1) = \int_{t_0}^{t_1} h(c(\tau))d\tau \qquad (6)$$

where h satisfies the conditions (a) and (b) in Lemma 1.

A further strength of the social norm approach is the flexibility of the axiomatic system. There are various attempts to reform the tax system towards a system where income taxation is partially substituted by a pollution tax[9]. In the axiomatic system, such an enlargement of the tax system is trivial, since (i) a third variable $p(t)$ besides the two basic variables wage and capital has to be defined, (ii) new axioms have to be added which reflect the political intentions related with the environment aspects and (iii) the old axiom system is enlarged to the case of three variables (for example the monotonicity axiom). Then, existence of the tax functional and the representation formula for this functional in this extended setup are derived in the same way as above.

2.4 Comparison with the Swiss tax law

We finally compare our axiomatic approach with the actual tax system for firms with tax domicile in the State of Zurich, Switzerland (for a detailed exposition see Hungerbühler and Vanini (2001)). By formalizing the tax law articles for income $i \in \mathbb{R}$ and capital $c \in \mathbb{R}$ one can derive an *implicit nonlinear* equation for the taxation function $T(i, c)$[10]. Although it is not a priori clear that such an equation has a unique solution $T(i, c)$ (both, non-existence and non-uniqueness of the tax function for specific values of wage and capital would be fatal for the execution of the law) one can prove that there always exists a unique solution $T(i, c)$ for any choice of income and capital. Nevertheless it turns out that $T(i, c)$ is not monotone in c. Therefore, the law allows for tax arbitrage. Furthermore the tax $T(i, c)$ is not continuous in the variables. Hence, a small change in the variables can have a jump increase in the tax payment. It actually turns out that the taxation function $T(i, c)$ induced by the actual law violates *all* axioms except the first one (positivity).

3 Discussion

We demonstrated in this paper that an axiomatic taxation model based on a choice of social norms can be formulated. The axioms reflect the following principles for the tax functional: Positivity,

[9] von Weizsäcker and Jesinghaus [1992], Pigou [1932], Jorgenson and Kun-Young [1990].

[10] Indeed the taxation law does not consider temporal aspects which leads to a tax function instead to a functional.

continuity in time, monotonicity in the variables income and wealth, economic incentive compatibility, time invariance and additivity. We proved that this system leads to an unique and explicitly given tax functional. Hence, the strong information assumptions in the classical optimal taxation models can be circumvented by using social norms. It was further indicated that the actual Swiss tax law violates all but one axiom.

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References

[1] Akerlof, G. and Yellen, J., "Fairness and Unemployment", American Economic Review (P+P) **78**, 1988, 44–49

[2] Cammerer, C. and Thaler, R., "Ultimatums, Dictators and Manners", Journal of Economic Perspectives **9**, 1995, 209–219

[3] Dasgupta, P. and Stiglitz, J. E., "On Optimal Taxation and Public Production", Review of Economic Studies **39**, 1972, 87–104

[4] Diamond, P. A. and Mirrless, J. A., "Optimal Taxation and Public Production I-II", American Economic Review **61**, 1971, 261–178

[5] Fehr, E. and Gächter, S., "Reciprocity and Economics: The Economic Implications of Homo Reciprocans", European Economic Review **42**, 1998, 845–859

[6] Guesnerie, R., "On the Direction of Tax Reform", Journal of Public Economics **7**, 1977, 179–202

[7] Güth, W., "An Evolutionary Approach to Explaining Cooperative Behavior by Reciprocal Incentives", International Journal of Game Theory **25**, 1995, 323–344

[8] Hahn, F. H., "On Optimal Taxation", Journal of Economic Theory **6**, 1973, 96–106

[9] Hungerbühler, N. and Vanini, P., "An Economic Analysis of Swiss Taxation Laws", Preprint, 2001

[10] Levine, D., "Fairness, Markets and Ability to Pay: Evidence from Compensation Executives", American Economic Review **83**, 1993, 1241–1259

[11] Mirrless, J. A., "The Theory of Optimal Taxation", In: K.J. Arrow and M. D. Intriligator (Eds.), Handbook of Mathematical Economics, North-Holland, Vol. III, 1991, 1197–1250

[12] Mussa, M., and Rosen, S., "Monopoly and Product Quality", Journal of Economic Theory **18**, 1978, 301–307

[13] Myerson, R., "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems", Journal of Mathematical Economics **10**, 1982, 67–81

[14] Pigou, A. C., "The Economics of Welfare", 4th ed., London, Macmillan, 1932

[15] Sheshinski, E., "The Optimal Linear Income Tax", Review of Economic Studies **39**, 1972, 179–208.

[16] Spence, M., "Non-Linear Prices and Economic Welfare", Journal of Public Economics **8**, 1977, 1–18.

[17] Roth, A., "Bargaining Experiments". In: Roth, A. and Kagel, J. (Eds.), Handbook of Experimental Economics, Princeton University Press, Princeton, 1995

[18] Vickrey, W., "Measuring Marginal Utility by Reactions to Risk", Econometrica **13**, 319–333

[19] Vickrey, W., "The Problem of Progression", University of Florida Law Review, 1968, 437–450