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Proof Without Words: Bijection Between Certain Lattice Paths

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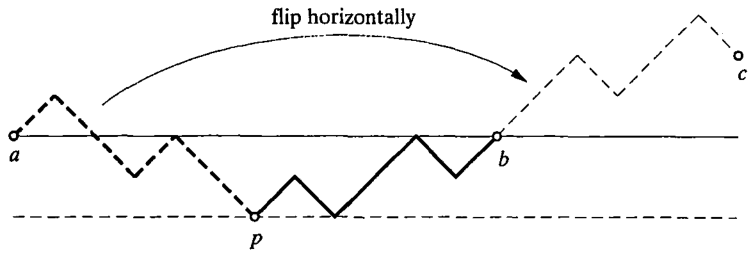
Dedicated to Ernst Specker on the occasion of his 78th birthday.



ab: lattice path with starting point and endpoint on the same (given) level
p: first minimum on the path ab
pc: lattice path staying above initial level (non-ruin path)

Conclusion There exist as many non-ruin paths of length $2n$ as paths of length $2n$ with starting point and endpoint on the same level, namely $\binom{2n}{n}$.

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An Antisymmetric Formula for Euler's Constant

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The formula

$$\gamma = \lim_{x \to 1^+} \sum_{n=1}^{\infty} \left(\frac{1}{n^x} - \frac{1}{x^n} \right) \tag{1}$$

shows that Euler's constant, γ, which is defined (see [1]) by

$$\gamma = \lim_{n \to \infty} \left(1 + \frac{1}{2} + \cdots + \frac{1}{n} - \log n \right), \tag{2}$$

is the limit as x approaches 1 from above of a series whose terms are antisymmetric in n and x. The formula also implies that γ is the limit as $x \to 1^+$ of the difference between the p-series $\sum_{n=1}^{\infty} 1/n^x$ and the geometric series $\sum_{n=1}^{\infty} 1/x^n$, because

$$\sum_{n=1}^{\infty} \left(\frac{1}{n^x} - \frac{1}{x^n} \right) = \sum_{n=1}^{\infty} \frac{1}{n^x} - \sum_{n=1}^{\infty} \frac{1}{x^n}$$

for $x > 1$. On the other hand, since the geometric series sums to $1/(x-1)$, the formula is itself an immediate consequence of the fact (see [4, Section 2.1]) that

$$\lim_{x \to 1} \left(\zeta(x) - \frac{1}{x-1} \right) = \gamma,$$

where $\zeta(x) = \sum_{n=1}^{\infty} 1/n^x$ is the Riemann zeta function. (For a connection between γ