Simple Simplex Sampling: Exploring the Solution Space of Degenerate Problems in High Dimensions

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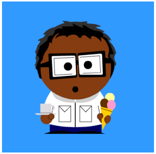

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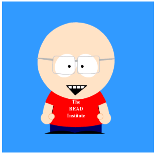

Justin Read

Andrea V. Maccò

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Outline

1. What is a Simplex?
2. Motivation: Gravitational Lensing
3. Simple Simplex Sampling
4. Estimating the Age of the Universe
5. Challenges

What is a Simplex?

Definition (Simplex)

A simplex is an n-dimensional polytope defined by $\mathbf{Ax} \leq \mathbf{b}$

- Simply connected
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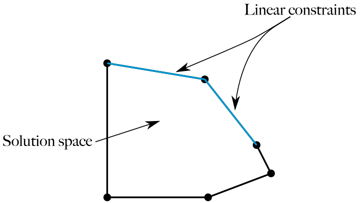

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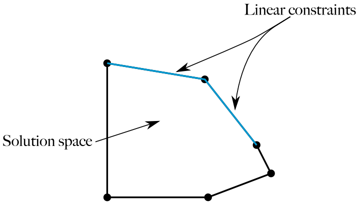
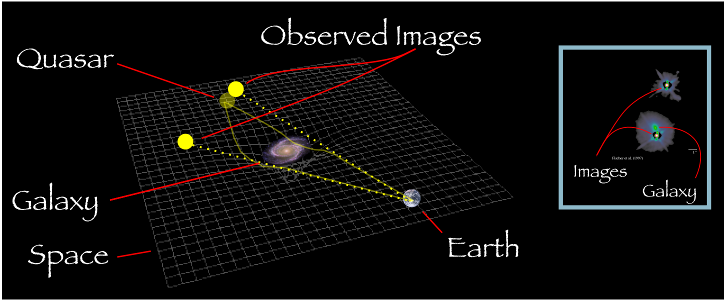


Definition (Linear Programming)

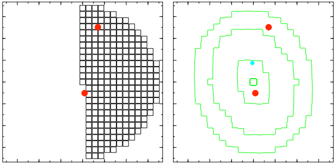
Maximize $\mathbf{c}^T\mathbf{x}$ such that
$\mathbf{Ax} \leq \mathbf{b}$ and $\mathbf{x} \geq 0$.

Motivation: Gravitational Lensing



GLs could tell us a lot about the universe (i.e., its age) and about the structure of galaxies, however...
The available data cannot constrain the exact distribution of the mass. It is these degeneracies we wish to explore.

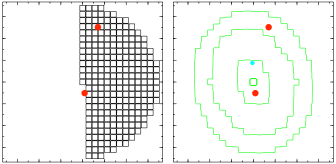
Motivation: Gravitational Lensing



The lens equation describes the arrival time of light at the observed position:

$$\tau(\theta) = \frac{1}{2}|\theta|^2 - \theta \cdot \beta - \int \ln|\theta - \theta'|\kappa(\theta')d^2\theta'$$

Motivation: Gravitational Lensing



We discretize the sky into grid cells and rewrite the equation as

$$\tau(\theta) = \frac{1}{2}|\theta|^2 - \theta \cdot \beta - \sum_n \kappa_n Q_n(\theta) + \text{shear terms}$$

Need to solve for each grid cell mass κ_n.

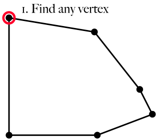
Motivation: Gravitational Lensing

- The κ_n are subject to other linear constaint equations that represent physical constraints as well as our own priors (e.g., smoothing, density gradients).
- Together these equations describe a simplex where the points within are solutions to the lens equation that perfectly match the input data. $N_{var} \sim 500$ and $N_{eq} \sim 1000$.
- We wish to sample these points to understand the range of solutions and the different kinds of degeneracies.

But how do we sample such a strange shape?

Simple Simplex Sampling

Use the simplex algorithm to find random vertice by maximizing randomly chosen objective functions.



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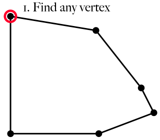

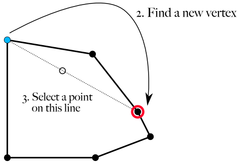

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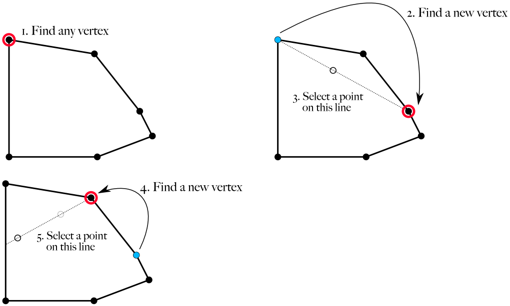

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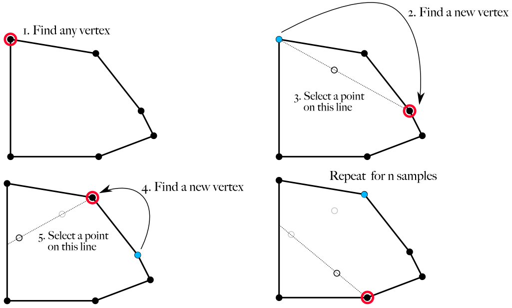

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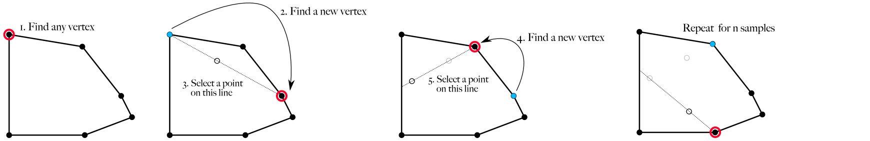

- Space has variables with different units.
  No understood metric.
- Non-intuitive choice of sample point solves two problems:

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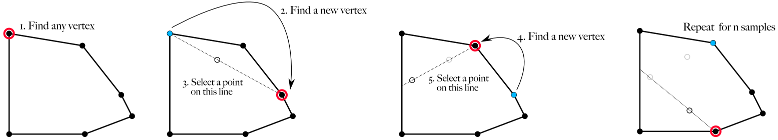

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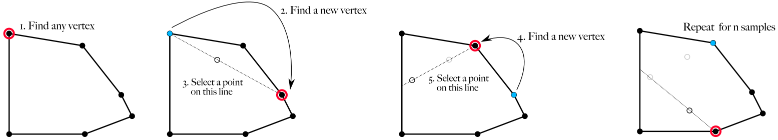

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Avoids bias when increasing resolution (pixel splitting).
No obvious metric on the space.

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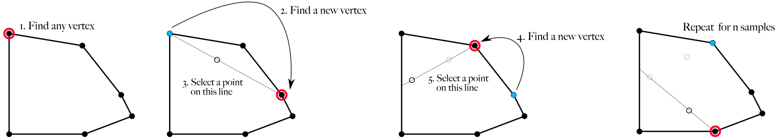

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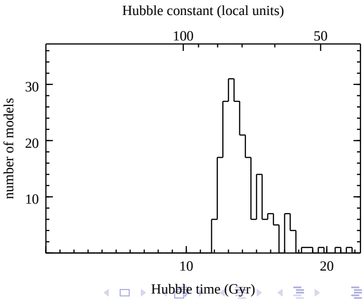
Should be insensitive to change of units.
For the same sequence of vertices, the same points will always be chosen.

Back to Lensing: Estimating the Age of the Universe

- The expansion rate of the universe H_0 is related to the observed time delay which is related to the mass distribution of the lensing galaxy.
- We model several lenses together. The latest paper used 11.
- A single point in the solution space corresponds to the values of parameters for all lenses simultaneously. H_0 is the same for all lenses at a given point.

- Since the mass distribution is degenerate we take many samples. Marginalizing over the other variables we find a posterior distribution function for H_0:



Challenges

1. Simplex Algorithm is slow, even when parallelized ($\sim O(n^3)$).
2. Is there a better way of randomly selecting a vertex without first enumerating them?