Part I: Math challenge

Dima Nikolenkov (presented by Kaloyan Slavov)

KS Trogen

dmnikolenkov@kst.ch

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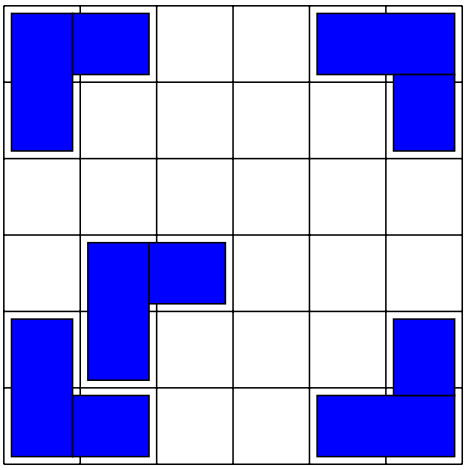
Math challenge at KS Trogen – format

- Participants: Kurzzeitgymnasium students, 14–16 years old
- 8 series, 4 problems each, distributed during the school year
- 2–3 weeks per problem set
- grade incentive
- No previous knowledge required (sometimes: prime factorization or divisibility rules)
- Focus: logical thinking, ability to find patterns, experimentation
- Difficulty: 2 problems are easy, 1 medium, and 1 requires a clear justification of the solution
- Article in the VSMP Bulletin, 131.

Examples – 1)



Cut a 6×6 square into L–corners of 3 unit squares each so that no two corners form a 2×3 rectangle.

Continue, in a unique way.

Examples – 2)

1			
	9		5
		14	

Is it possible to arrange 8 of the 9 numbers

$$2, 3, 4, 7, 10, 11, 12, 13, 15 \rightarrow \textit{sum} = 77$$

in the vacant squares of the table so that the arithmetic average of the numbers in each row and each column is the same integer? Find all such arrangements.

Sketch. The sum along each row should be a multiple of 4
$\Longrightarrow$ the sum of all written numbers is a multiple of 4.
The sum along each column should be a multiple of 3
$\Longrightarrow$ the sum of all written numbers is a multiple of 3.
$\Longrightarrow$ $12 \mid (29 + 77 - x) \Longrightarrow x = 10$.

Examples – 3)

Poseidon has octopuses with $6, 7$, and 8 legs:

- 7-legged octopuses always lie;
- 6 and 8-legged octopuses always tell the truth.

Poseidon gets a blue, a green, a yellow, and a red octopus to answer the question:

How many legs do you have altogether?

— blue: 28 — green: 27 — yellow: 26 — red: 25

Figure out how many legs each of them has.

Answer: $7, 6, 7, 7$, respectively.

Dima's invitation

- Dima Nikolenkov, `dmnikolenkov@kst.ch`
- Math Challenge at KS Trogen has not started yet this year.
- You are invited to join!
- Dima can provide problem series **with solutions.**
- Grading should take about an hour for 15 students.

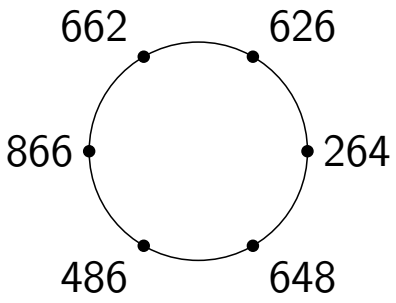
An exploration problem

For a given positive integer n, we wish to construct a circle of six numbers as shown at right so that the circle has the following properties:

a) The six numbers are different three-digit numbers, none of whose digits is a 0.

b) Going around the circle clockwise, the first two digits of each number are the last two digits, in the same order, of the previous number.

c) All six numbers are divisible by n.



An example with $n = 2$.

For which $n = 2, 3, \ldots, 9$ does such a circle exist?