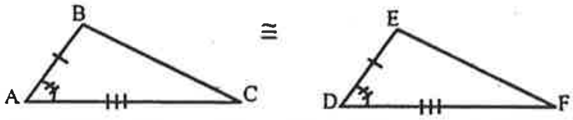
6. Congruent triangles

Triangle congruency can be determined in <u>SIX</u> ways:

1) SSS – (side – side – side)

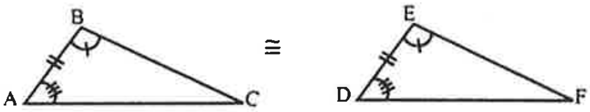
If AB = DE , AC = DF , BC = EF, then ΔABC ≅ ΔDEF by SSS

2) SAS = (side – angle – side)



If AB = DE , ∠A = ∠D , AC = DF, then ΔABC ≅ ΔDEF by SAS

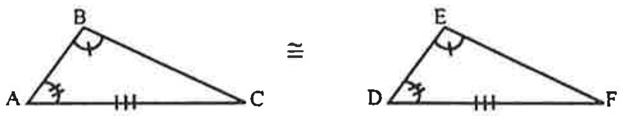
3) ASA – (angle – side – angle)



If ∠A = ∠D , AB = DE , ∠B = ∠E, then ΔABC ≅ ΔDEF by ASA

4) AAS = (angle – angle – side)

**do not get ASA and AAS mixed up. In ASA the side is between the two given angles. In AAS the side is not between the two given angles.*



If ∠A = ∠D , ∠B = ∠E , AC = DF, then ΔABC ≅ ΔDEF by AAS

5) HL – (hypotenuse – leg) **the triangle must be a right triangle*

If AC = DF , AB = DE (in a right triangle), then ΔABC ≅ ΔDEF by HL

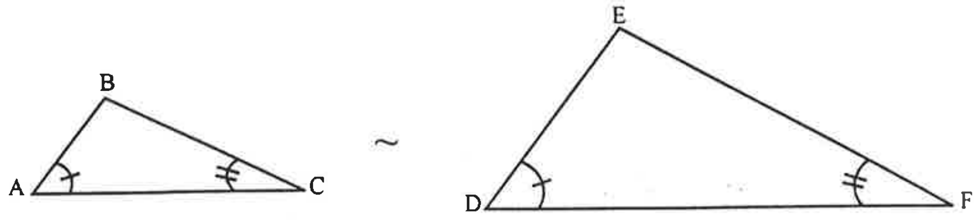
6) HA – (hypotenuse – angle) **the triangle must be a right triangle*

If AC = DF , ∠A = ∠D (in a right triangle), then ΔABC ≅ ΔDEF by HA

Note: When you have a triangle congruent to another triangle, then and only then can you say the triangles have three equal angles and three equal sides. The reason for making any of these six statements is CPCTC (corresponding parts of congruent triangles are congruent).

7. Similar triangles

a) If two angles of one triangle equal two angles of another triangle, then the triangles are similar.



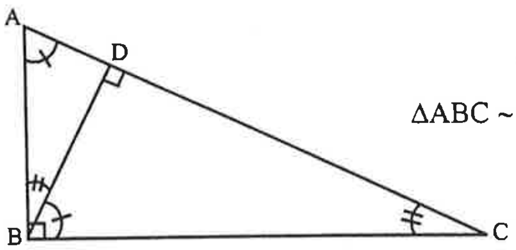
If $\angle A = \angle D$ and $\angle C = \angle F$, then $\Delta ABC \sim \Delta DEF$ by AA or AAA
(*Remember, if two angles are equal, the third angle is also equal by third angle of a triangle*)

b) Similar triangles have sides that are proportional.

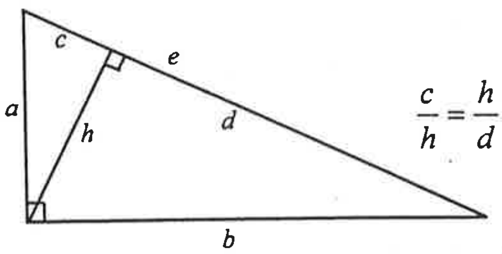
If $\Delta ABC \sim \Delta DEF$, then $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$.

b) Similarity properties in right triangles

The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.



$\Delta ABC \sim \Delta BDC \sim \Delta ADB$

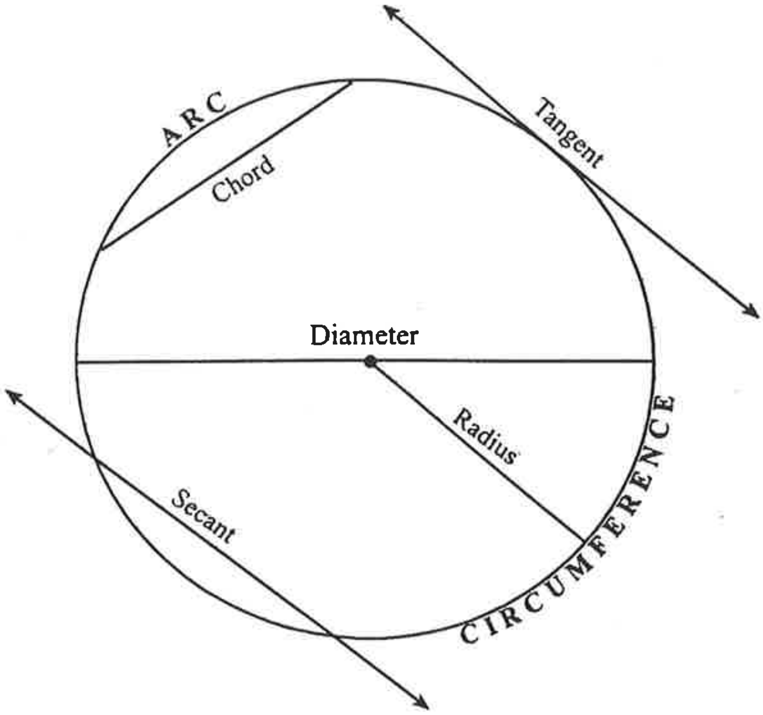


$\dfrac{c}{h} = \dfrac{h}{d}$, $\dfrac{e}{a} = \dfrac{a}{c}$, $\dfrac{e}{b} = \dfrac{b}{d}$

8. Definitions

- **Bisector of a segment** – the midpoint of a line segment.
- **Bisector of an angle** – the ray in the interior of an angle that divides the angle into two equal angles.
- **Median of a triangle** – the segment connecting a vertex of the triangle to the midpoint of the opposite side.
- **Perpendicular** – two rays, segments or lines such that the lines containing them form a right angle. The symbol $\perp$ means "is perpendicular to."

9. Circle terminology.



- **Circumference** – the perimeter of a circle.
- **Arc** – a part of a circle connecting two points on the circle.
- **Radius** – a segment connecting the centre of a circle with a point on the circle.
- **Diameter** – a segment connecting two points on the circle and containing the centre of the circle.
- **Chord** – a segment whose endpoints are on a given circle.
- **Secant** – a line that intersects the circle in two points.
- **Tangent** – a line that intersects the circle in exactly one point.

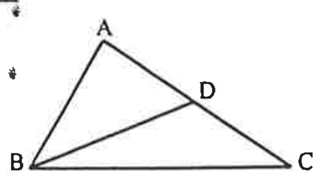
10. Circle formulas.

Circumference of a circle: $C = 2\pi r = \pi d$
($r =$ radius, $d =$ diameter)

Area of a circle: $A = \pi r^2$

Draw a conclusion from each statement. Your conclusion should be based only on the data that is given.

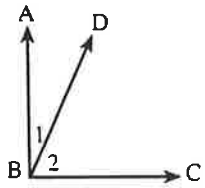
Example:



Given: $\overrightarrow{BD}$ is a median

Conclusion: AD = DC

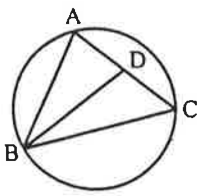
1.



Given: ∠1 and ∠2 are complementary angles

Conclusion: ______________________

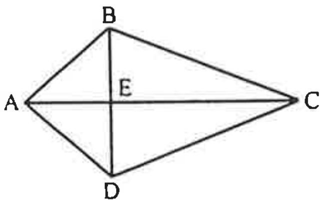
2.



Given: BD bisects AC

Conclusion: ______________________

3.

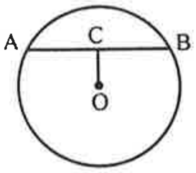


Given: AC ⊥ bisector of BD

Conclusion (give two) ______________________

____

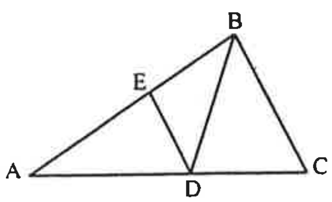
4.



Given: C is the midpoint of AB

Conclusion: ______________________

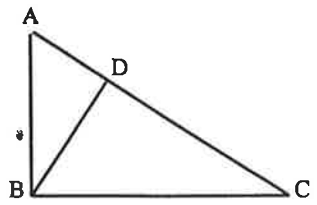
5.



Given: ∠ABC is a right angle

Conclusion: ______________________

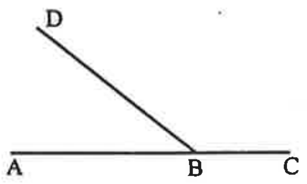
6.



Given: AC ⊥ BD

Conclusion: ______________________

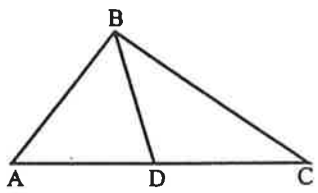
7.



Given: ∠ABD and ∠DBC are supplementary angles

Conclusion: ______________________

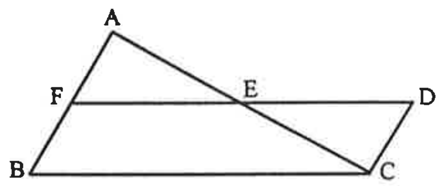
8.



Given: AD = DC

Conclusion: ______________________

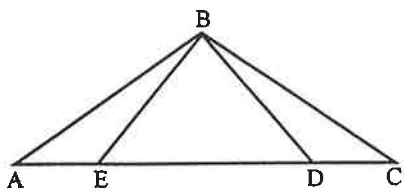
9.



Given: AC bisects FD

Conclusion: ______________________

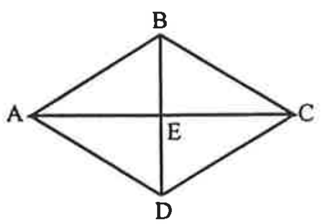
10.



Given: ∠ABD = ∠CBE

Conclusion: ______________________

11.

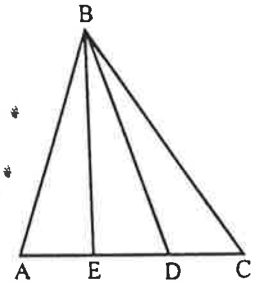


Given: AC and BD bisect each other

Conclusion: (give two) ______________________

____

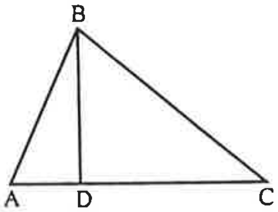
C 12.



Given: E and D trisect AC

Conclusion: ____________________________

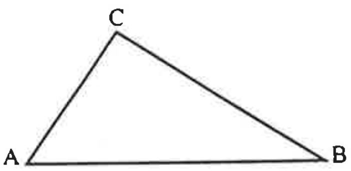
C 13.



Given: BD is an altitude of ΔABC

Conclusion: ____________________________

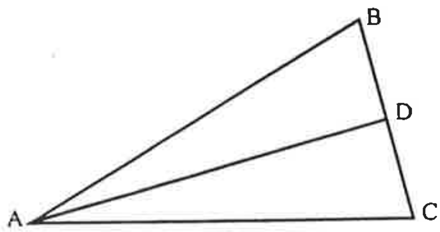
C 14.



Given: $\angle A = 57°$ and $\angle B = 33°$

Conclusion: ____________________________

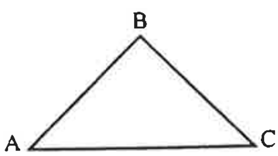
C 15.



Given: $BD = DC$ and $\angle ADB = \angle ADC$

Conclusion: ____________________________

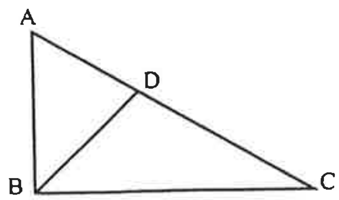
C 16.



Given: $\angle A \cong \angle C$

Conclusion: ____________________________

C 17.

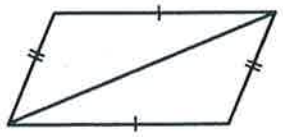


Given: $\angle ABD$ and $\angle DBC$ are complementary angles

Conclusion: ____________________________

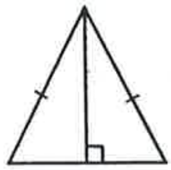
Triangle congruency – Are the following pairs of triangles congruent? If congruent, state one of the following six congruencies: SSS, SAS, ASA, AAS, HL, HA.

D 1.



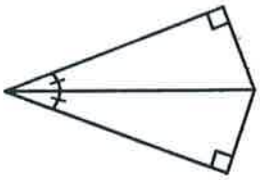
______

D 2.



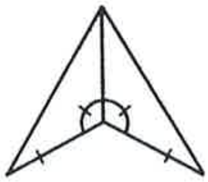
______

D 3.



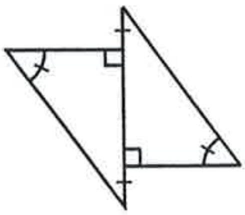
______

D 4.



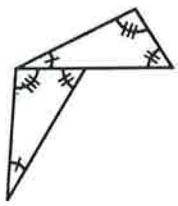
______

D 5.



______

D 6.



______

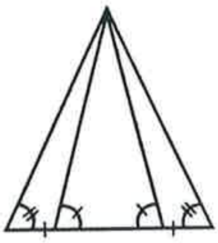
D 7.

______

D 8.

______

D 9.

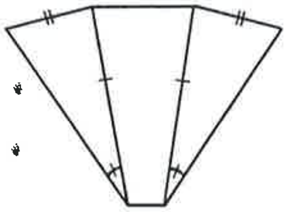


______

D 10.

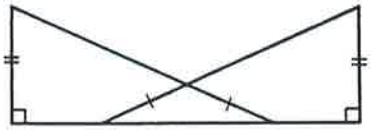
______

D 11.



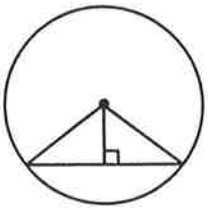
____

D 12.



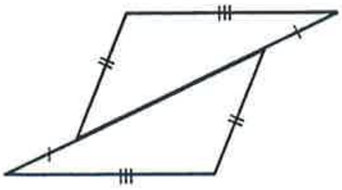
____

D 13.



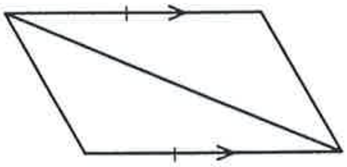
____

D 14.



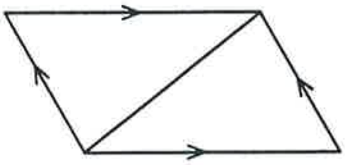
____

D 15.



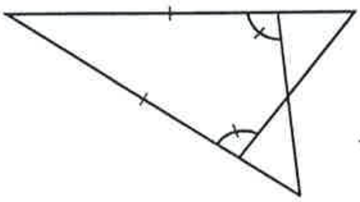
____

D 16.



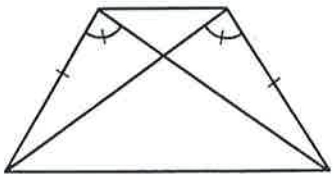
____

D 17.



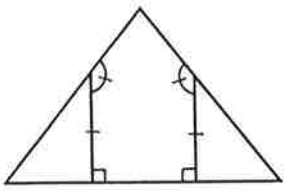
____

D 18.



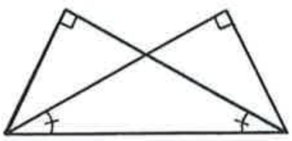
____

D 19.



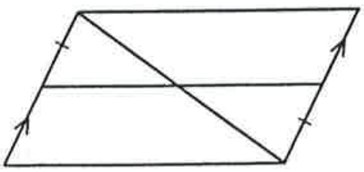
____

D 20.



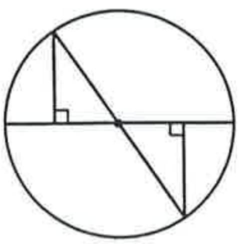
____

D 21.



____

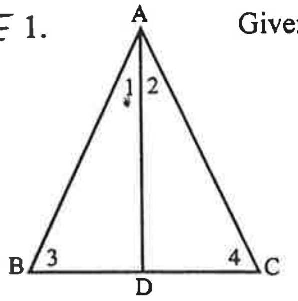
D 22.



____

PROOFS

E 1.

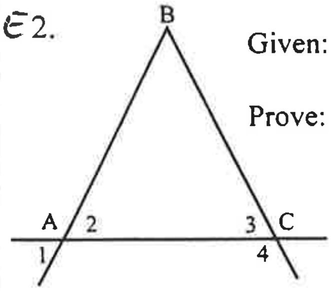


Given: ∠3 is complementary to ∠1
: ∠4 is complementary to ∠2
: AD bisects ∠BAC

Prove: ∠3 = ∠4

Proof

E 2.

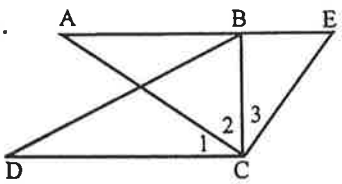


Given: ∠1 is supplementary to ∠4

Prove: ∠2 = ∠3

Proof

E 3.

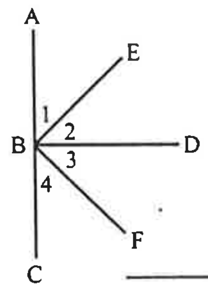


Given: BC ⊥ CD
: AC ⊥ CE

Prove: ∠1 = ∠3

Proof

E 4.

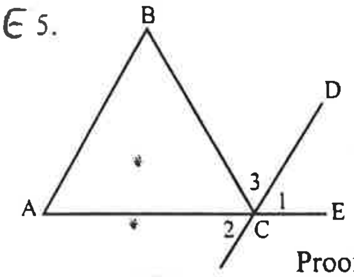


Given: AC ⊥BD
: BD bisects ∠EBF

Prove: ∠1 = ∠4

Proof

5.

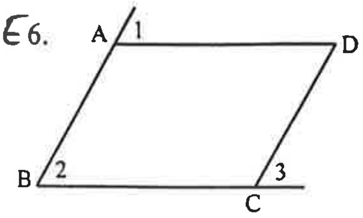


Given: ∠2 = ∠3

Prove: $\overrightarrow{CD}$ bisects ∠BCE

Proof

6.

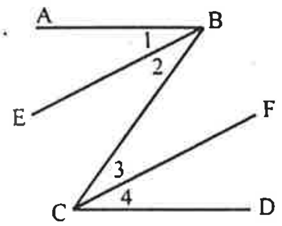


Given: ∠1 = ∠3
: AB || CD

Prove: AD || BC

Proof

7.

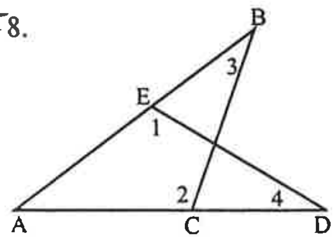


Given: AB || CD
: $\overrightarrow{BE}$ bisects ∠ABC
: $\overrightarrow{CF}$ bisects ∠BCD

Prove: ∠2 = ∠3

Proof

8.

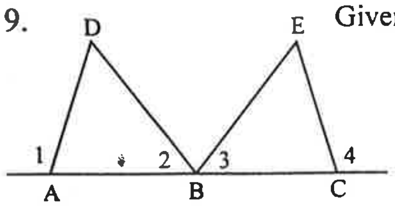


Given: ∠1 = ∠2

Prove: ∠3 = ∠4

Proof

Ɛ 9.

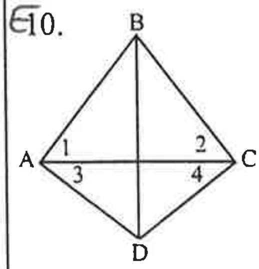


Given: B is the midpoint of AC
: ∠1 = ∠4
: ∠2 = ∠3

Prove: ΔADB ≅ ΔCEB

Proof

Ɛ10.

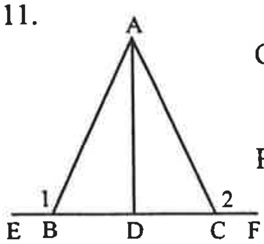


Given: ∠1 = ∠2
: ∠3 = ∠4

Prove: ΔABD ≅ ΔCBD

Proof

Ɛ 11.

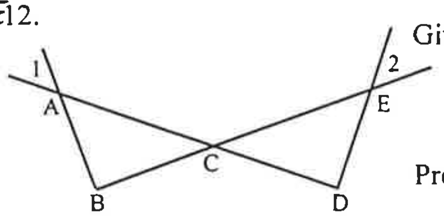


Given: AD is the ⊥ bisector of BC

Prove: ∠1 = ∠2

Proof

Ɛ12.

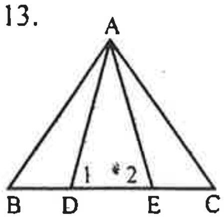


Given: AB ⊥ BE
: AD ⊥ DE
: BC = CD

Prove: ∠1 = ∠2

Proof

E 13.

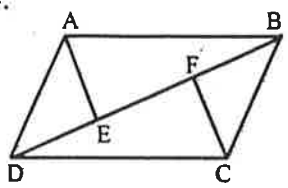


Given: AD = AE
: BE = DC
: ∠1 = ∠2

Prove: ΔABD ≅ ΔACE

Proof

E 14.

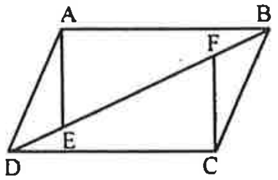


Given: AD || BC
: AD = BC
: AE ⊥ DB
: CF ⊥ DB

Prove: ΔADE ≅ ΔCBF

Proof

E 15.

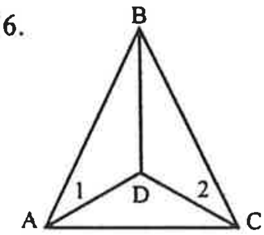


Given: AE ⊥ AB
: CF ⊥ DC
: AE = CF
: DE = BF

Prove: AB = DC

Proof

E 16.

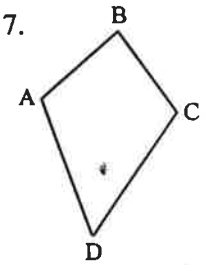


Given: ∠1 = ∠2
: AB = CB

Prove: AD = CD

Proof

17.

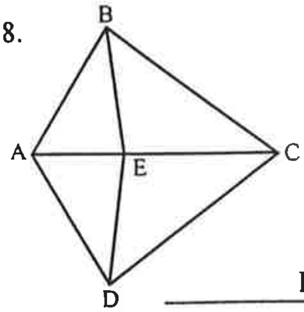


Given: AB = CB
: AD = CD

Prove: ∠A = ∠C

Proof

18.

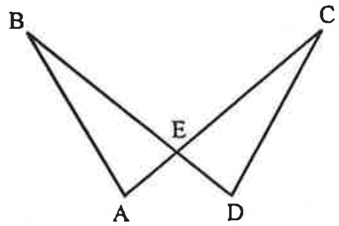


Given: BC = DC
: BE = DE

Prove: AB = AD

Proof

19.

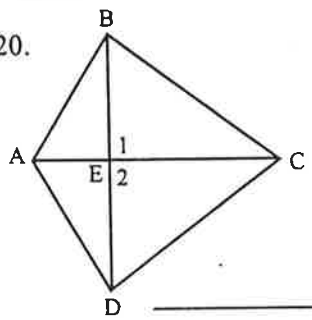


Given: AB = CD
: AC = BD

Prove: ∠A = ∠D

Proof

20.



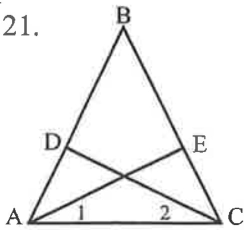
Given: AB = AD
: BC = DC

Prove: AE ⊥ bisector of BD

Proof

Overlapping Triangles

E21.

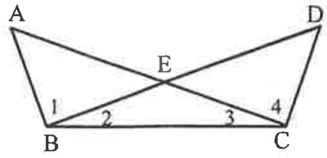


Given: CD ⊥ AB
: AE ⊥ BC
: ∠1 = ∠2

Prove: ΔAEC ≅ Δ CDA

Proof

E22.

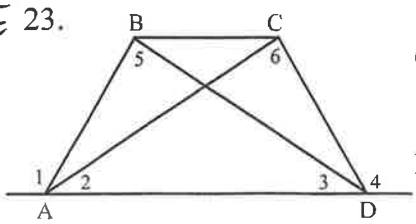


Given: ∠1 = ∠4
: ∠2 = ∠3

Prove: ∠A = ∠D

Proof

E 23.

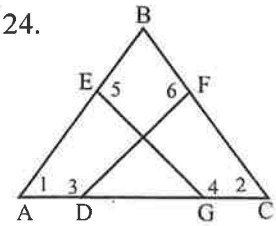


Given: ∠1 = ∠4
: ∠2 = ∠3

Prove: ∠5 = ∠6

Proof

E 24.



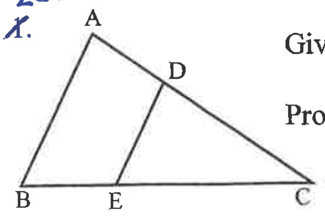
Given: AE = CF
: ∠1 = ∠2
: ∠3 = ∠4

Prove: ∠5 = ∠6

Proof

Similar Triangles

E 25. 1.

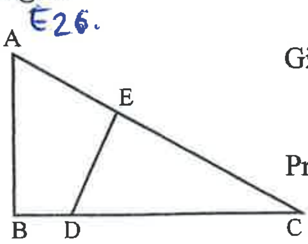


Given: AB ∥ DE

Prove: ΔABC ~ ΔDEC

Proof

E 26. 2.

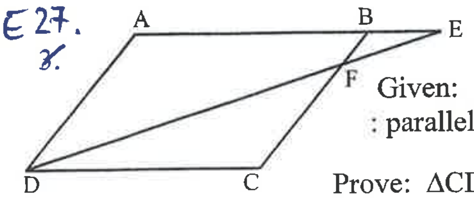


Given: AB ⊥ BC
: DE ⊥ AC

Prove: ΔABC ~ ΔDEC

Proof

E 27. 3.

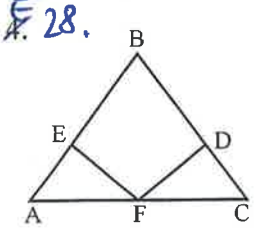


Given: $\overline{ABE}$
: parallelogram ABCD

Prove: ΔCDF ~ ΔBEF

Proof

E 28. 4.



Given: AB = BC
: EF ⊥ AB
: FD ⊥ BC

Prove: ΔAEF ~ ΔDFC

Proof

