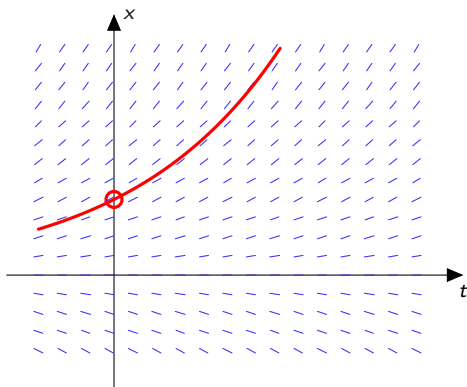
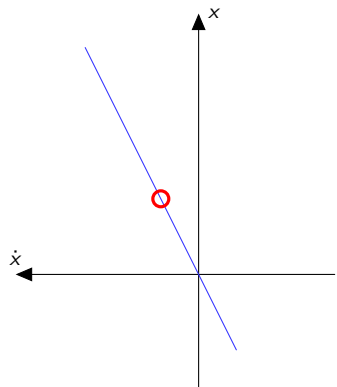
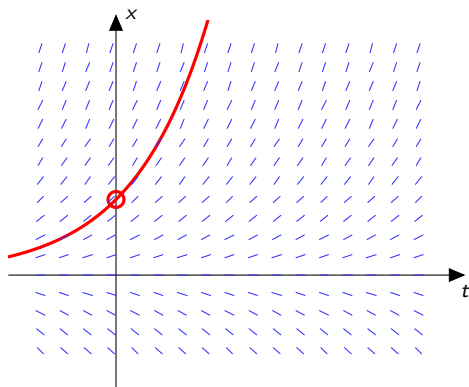
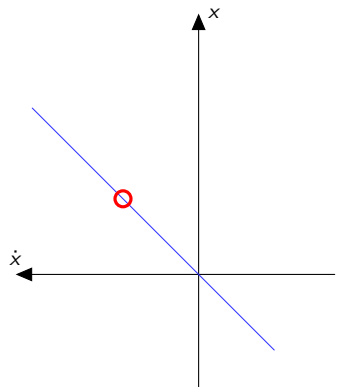


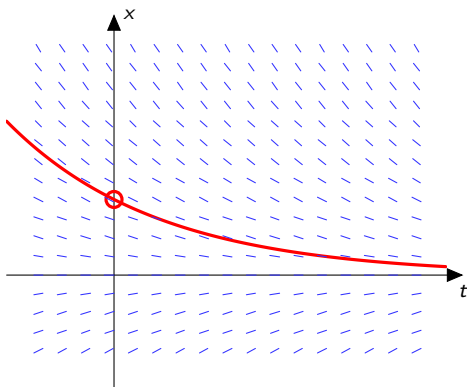
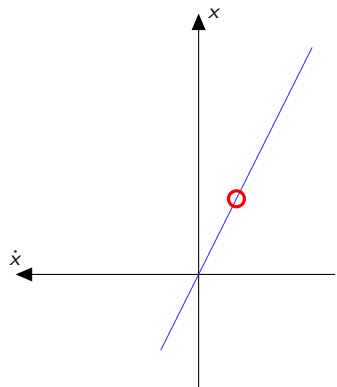
Die Differentialgleichung $\dot{x} = kx$ mit Wachstumsfaktor k



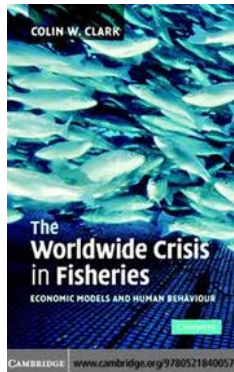
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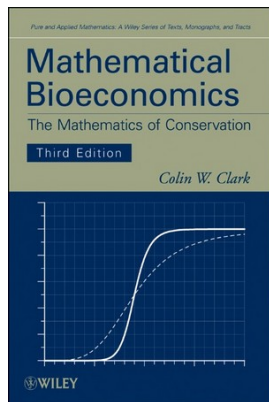
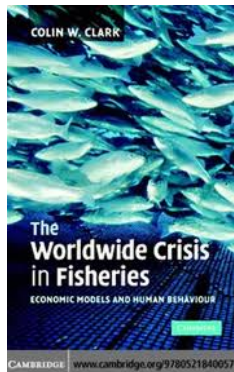
$\dot{x} = kx$ mit negativem Wachstumsfaktor k



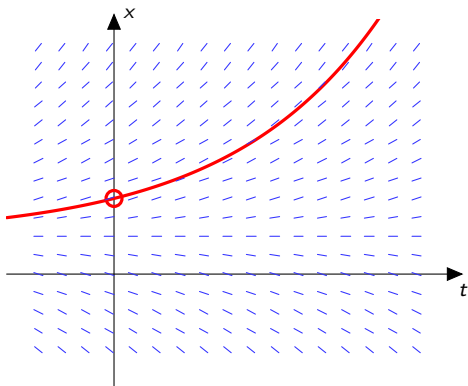
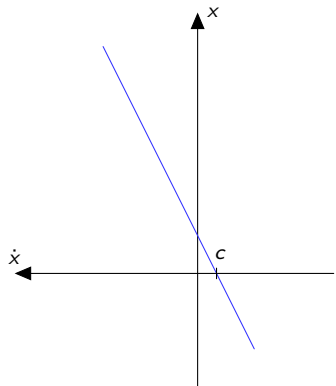
Fischpopulationen und Fangquoten



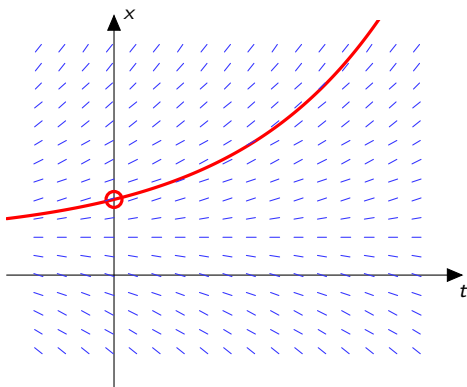
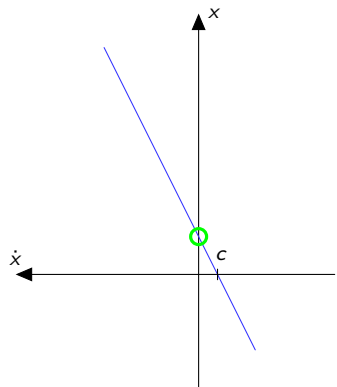
Fischpopulationen und Fangquoten



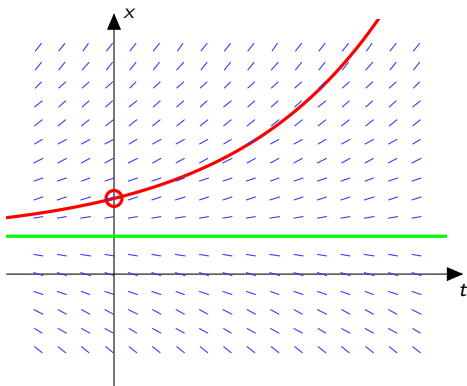
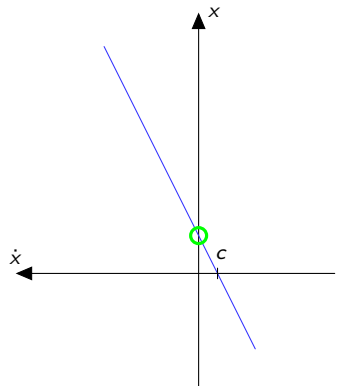
$$\dot{x} = kx - c \text{ mit Fangquote } c$$



$$\dot{x} = kx - c \text{ mit Fangquote } c$$



$\dot{x} = kx - c$ mit Fangquote c



Verhulst und Schwellenwerte



Pierre-Francois Verhulst
(1804 – 1849)

Die logistische Gleichung:

$$\dot{x} = K(x)x$$

Verhulst und Schwellenwerte



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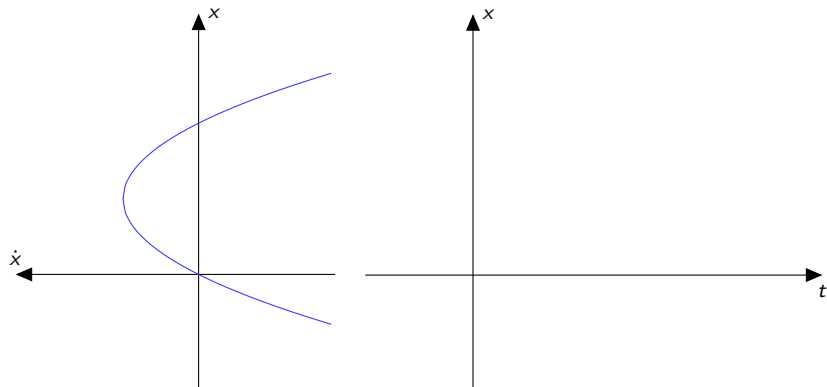
$$\dot{x} = K(x)x$$

$$\dot{x} = k \left(1 - \frac{x}{s} \right) x$$

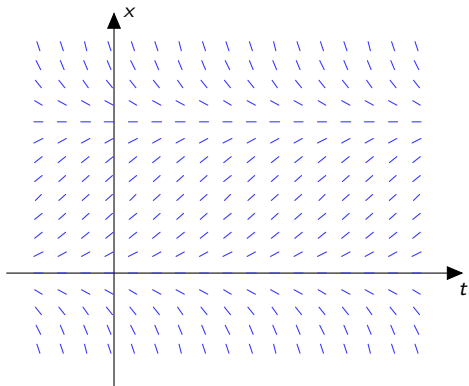
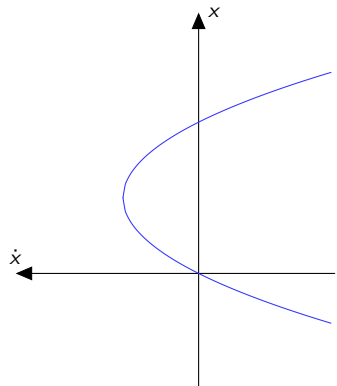
k = intrinsischer Wachstumsfaktor

s = Schwellenwert

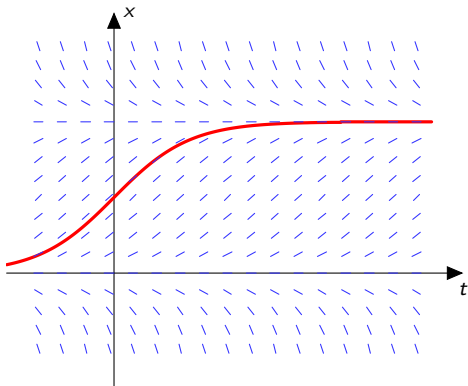
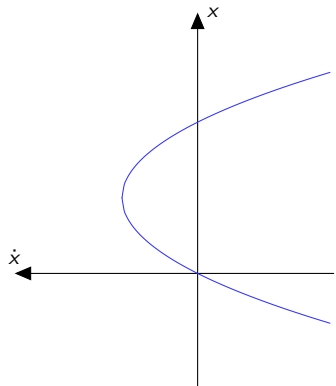
$$\dot{x} = k\left(1 - \frac{x}{s}\right)x \text{ mit Sättigungsfaktor } 1 - \frac{x}{s}$$



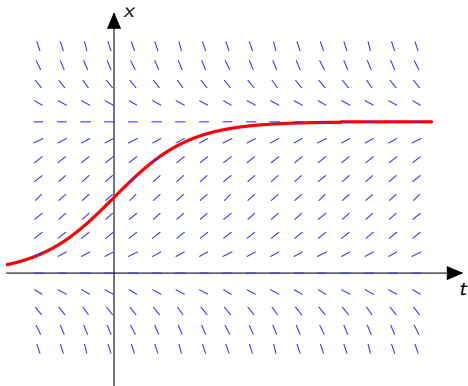
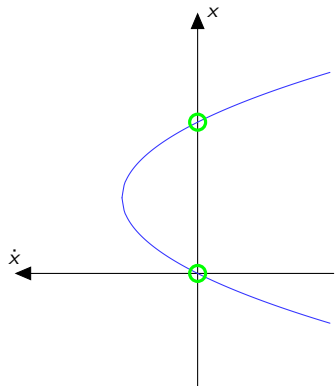
$$\dot{x} = k\left(1 - \frac{x}{s}\right)x \text{ mit Sättigungsfaktor } 1 - \frac{x}{s}$$



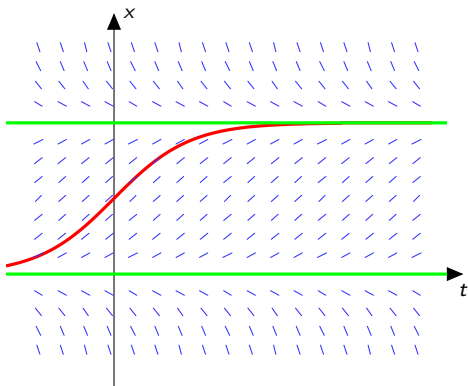
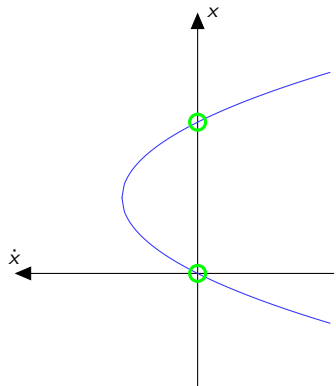
$$\dot{x} = k\left(1 - \frac{x}{s}\right)x \text{ mit Sättigungsfaktor } 1 - \frac{x}{s}$$



$$\dot{x} = k\left(1 - \frac{x}{s}\right)x \text{ mit Sättigungsfaktor } 1 - \frac{x}{s}$$



$$\dot{x} = k\left(1 - \frac{x}{s}\right)x \text{ mit Sättigungsfaktor } 1 - \frac{x}{s}$$



Verhulst: Daten aus England

In: Corresp. Math. Phys. 10, 1838, S. 113 - 121



N^o 3.

Tableau des progrès de la population dans le comté d'Essex en Angleterre, depuis 1811 jusqu'à 1831.

ANNÉES.	D'APRÈS les registres des paroisses.	D'APRÈS la formule.	ERREUR proportionnelle.
1811	252,473	252,473	0,000
1812	255,410	256,600	+ 0,004
1813	258,383	260,500	+ 0,006
1814	262,705	264,400	+ 0,006
1815	268,143	268,300	+ 0,008
1816	270,270	272,100	+ 0,006
1817	274,088	275,650	+ 0,005
1818	278,513	279,300	+ 0,002
1819	282,232	282,700	+ 0,001
1820	285,787	286,100	+ 0,001
1821	289,424	289,424	0,000
1822	292,085	292,600	- 0,001
1823	296,436	295,750	- 0,002
1824	299,166	298,800	- 0,001
1825	302,302	301,600	- 0,002
1826	304,482	304,570	0,000
1827	306,474	307,300	+ 0,002
1828	308,887	309,800	+ 0,003
1829	311,807	312,400	+ 0,002
1830	314,306	314,900	+ 0,002
1831	317,233	317,233	0,000

Verhulst: Daten aus Belgien



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CORRESPONDANCE

N^o 2.

Population de la Belgique.

ANNÉES.	D'APRÈS L'ÉTAT CIVIL.	D'APRÈS LA FORMULE.	ERREUR.
1815	3,494,965 33,485	3,494,985 35,315	0,000
1816	3,628,460 38,104	3,630,300 35,500	0,000
1817	3,666,554 15,329	3,665,800 35,500	0,000
1818	3,681,883 23,708	3,681,300 35,600	+ 0,005
1819	3,608,691 37,303	3,639,000 35,700	+ 0,008
1820	3,645,894 30,774	3,672,600 36,800	+ 0,007
1821	3,676,688 46,200	3,708,400 36,800	+ 0,008
1822	3,721,888 47,868	3,744,200 36,000	+ 0,006
1823	3,769,726 46,523	3,780,200 36,049	+ 0,003
1824	3,816,349 60,828	3,816,249 36,060	0,000
1825	3,867,077 46,780	3,862,299 36,001	- 0,004
1826	3,913,857 42,661	3,888,300 36,100	- 0,006
1827	3,956,618 38,462	3,924,400 36,200	- 0,008
1828	3,994,080 46,619	3,980,600 36,300	- 0,008
1829	4,041,489 53,213	3,996,800 36,300	- 0,011
1830	4,074,712 22,178	4,033,100 36,300	- 0,010
1831	4,099,890 33,231	4,069,400 36,400	- 0,007
1832	4,130,121 12,136	4,105,800 36,457	- 0,006
1833 1 ^{er} janvier.	4,142,267	4,142,267	0,000

Verhulst: Daten aus Russland



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CORRESPONDANCE

N^o 4.

Population de la Russie. (Individus de la communion grecque.)

ANNÉES.	D'APRÈS LES REGISTRES DE l'église grecque.	D'APRÈS LA FORMULE.
1796	29,177,980 461,521	29,177,980
1797	29,639,501 461,525	30,332,000
1798	30,101,026 428,248	31,424,000
1799	30,629,274 432,418	32,466,000
1800	30,961,692 440,000	33,360,000
1801	31,401,692 453,205	34,338,000
1802	31,854,897 616,097	35,191,000
1803	32,470,994 476,372	35,988,000
1804	32,946,366 568,489	36,731,000
1805	33,514,835 542,701	37,423,000
1806	34,057,536 500,693	38,065,000
1807	34,668,198 468,508	38,661,000
1808	35,029,706 462,478	39,213,000
1809	35,489,184 466,712	39,723,000
1810	35,955,896 471,546	40,195,000

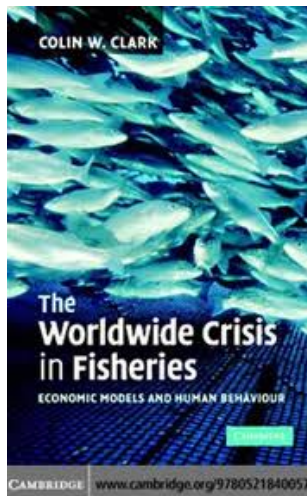
Verhulst: Nochmals Daten aus Belgien (1847)



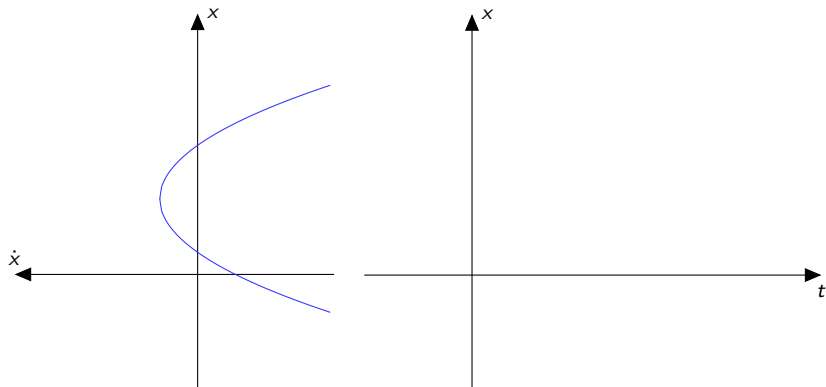
ANNÉES.	POPULATION observée.	POPULATION calculée.	ANNÉES.	POPULATION observée.	POPULATION calculée.
1815.....	5,027,255	5,027,500	1853.....	4,404,220	4,458,600
à	507,602	420,900		44,540	57,400
1825.....	4,034,855	4,048,200	1856.....	4,448,709	4,476,000
	48,890	40,400		45,910	57,100
1826.....	4,075,751	4,088,600	1857.....	4,494,688	4,515,100
	45,089	40,000		50,909	56,800
1827.....	4,118,840	4,128,000	1858.....	4,525,687	4,540,900
	41,459	59,000		45,021	56,500
1828.....	4,160,279	4,168,500	1859.....	4,570,708	4,580,400
	40,840	59,400		58,068	56,900
1829.....	4,210,128	4,207,900	1840.....	4,608,776	4,622,600
	50,985	59,200		41,024	55,900
1850.....	4,247,115	4,247,100	1841.....	4,650,400	4,658,500
	*58,856	58,900		42,790	55,600
1851.....	4,285,969	4,286,000	1842.....	4,695,190	4,694,100
	40,728	58,600		54,201	55,500
1852.....	4,526,097	4,524,600	1845.....	4,727,591	4,729,400
	19,245	58,500		55,855	55,000
1855.....	4,545,940	4,562,900	1844.....	4,765,246	4,764,400
	50,274	58,000		*37,615	54,700
1854.....	4,576,214	4,400,900	1845.....	4,800,861	4,799,100
	28,006	57,700			

NB. Les nombres marqués d'un astérisque sont hypothétiques.

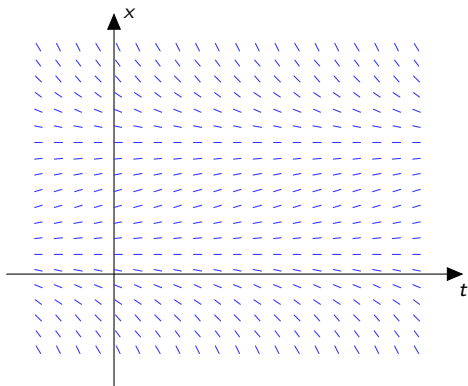
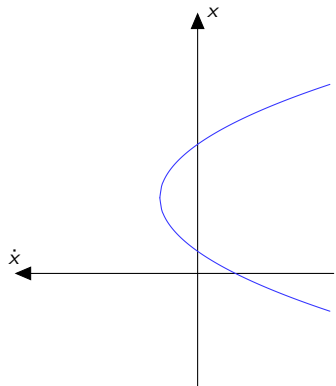
Fischpopulationen und Fangquoten



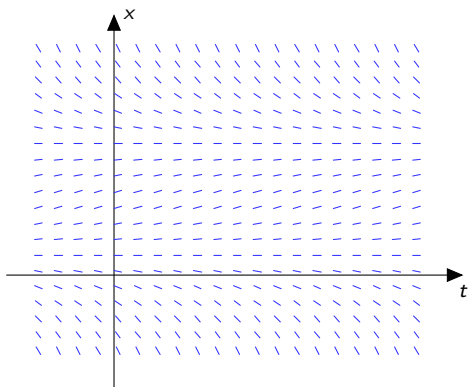
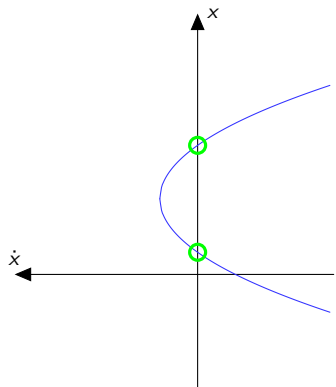
$$\dot{x} = k\left(1 - \frac{x}{s}\right)x - c \text{ mit konstanter Fangquote } c$$



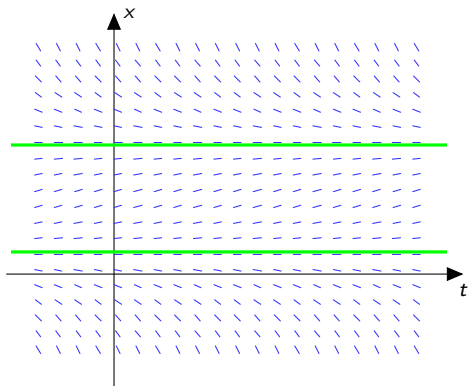
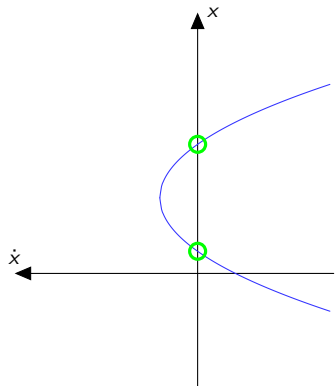
$\dot{x} = k(1 - \frac{x}{s})x - c$ mit konstanter Fangquote c



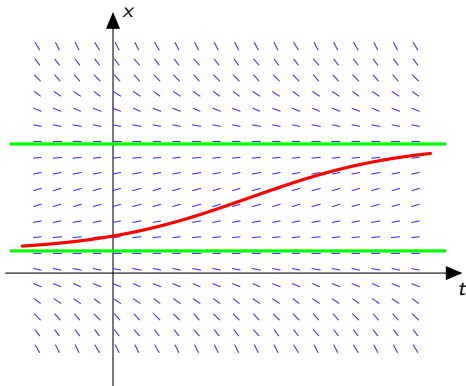
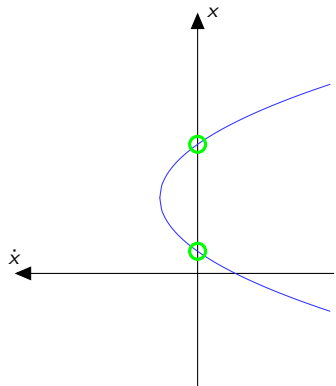
$\dot{x} = k(1 - \frac{x}{s})x - c$ mit konstanter Fangquote c



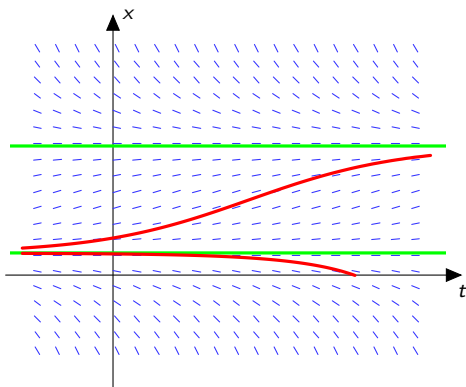
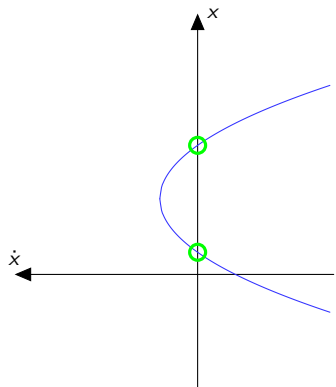
$\dot{x} = k(1 - \frac{x}{s})x - c$ mit konstanter Fangquote c



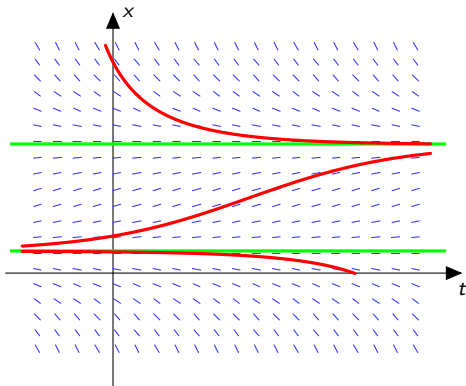
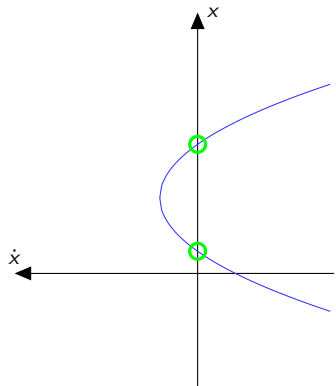
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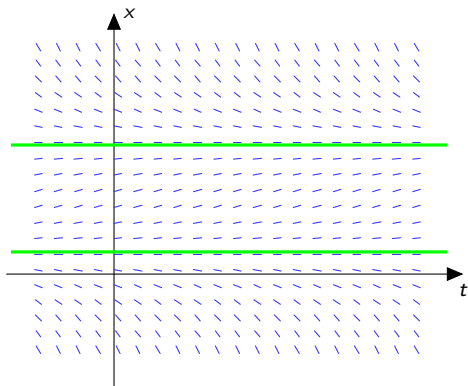
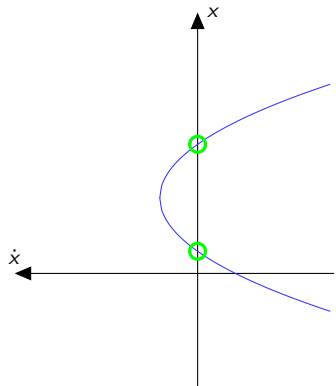
$$\dot{x} = k\left(1 - \frac{x}{s}\right)x - c \text{ mit konstanter Fangquote } c$$



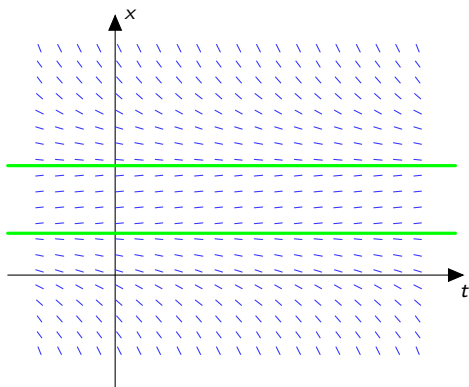
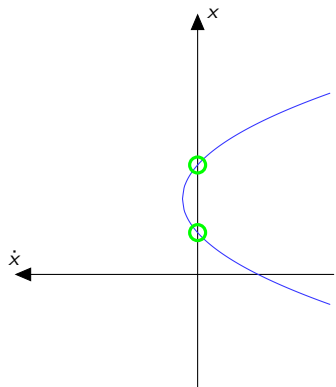
$$\dot{x} = k\left(1 - \frac{x}{s}\right)x - c \text{ mit konstanter Fangquote } c$$



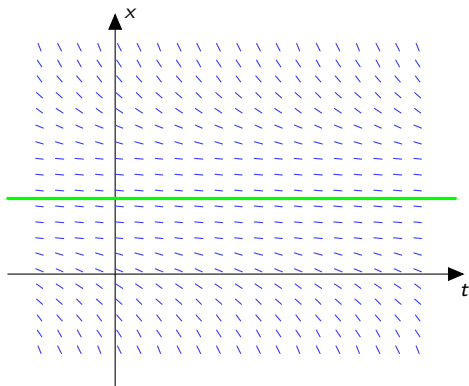
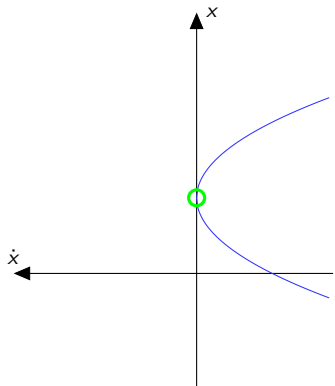
$\dot{x} = k(1 - \frac{x}{s})x - c$ mit konstanter Fangquote c



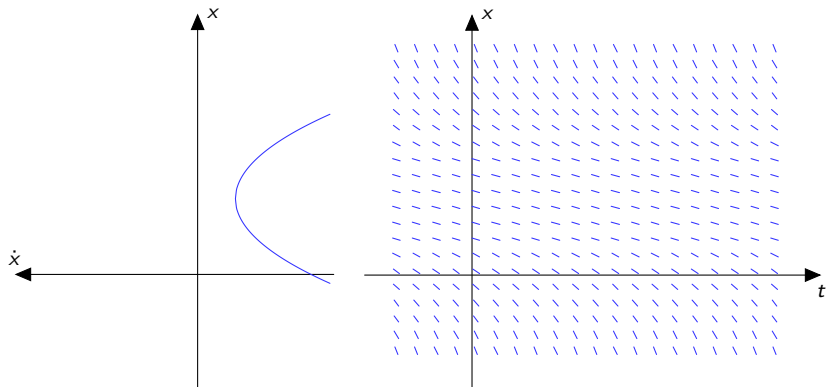
$$\dot{x} = k\left(1 - \frac{x}{s}\right)x - c \text{ mit konstanter Fangquote } c$$



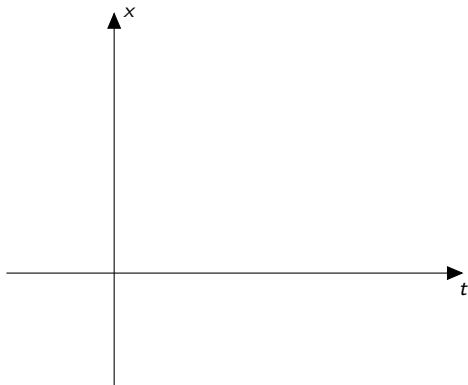
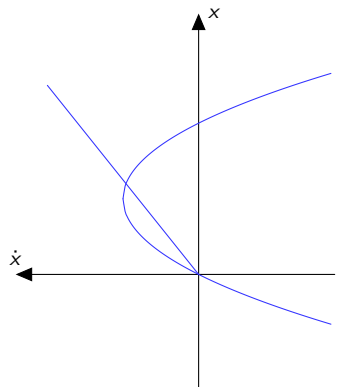
$$\dot{x} = k\left(1 - \frac{x}{s}\right)x - c \text{ mit konstanter Fangquote } c$$



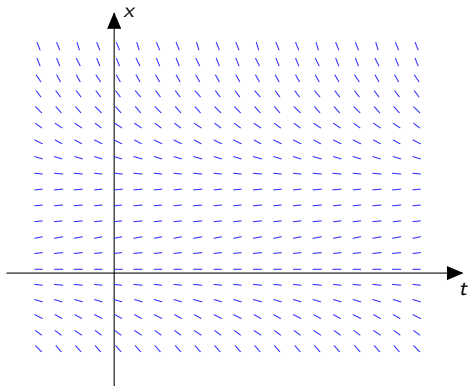
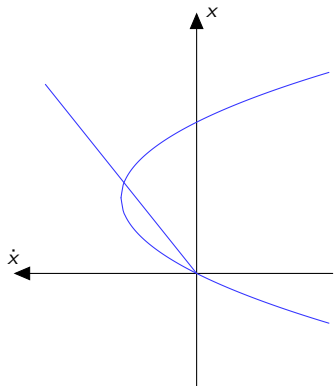
$$\dot{x} = k\left(1 - \frac{x}{s}\right)x - c \text{ mit konstanter Fangquote } c$$



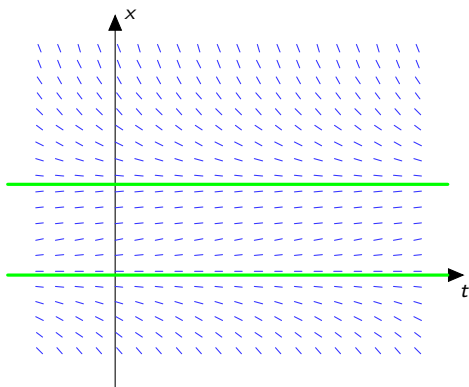
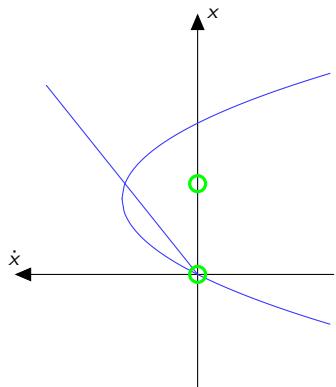
$$\dot{x} = k\left(1 - \frac{x}{s}\right)x - cx \text{ mit relativer Fangquote } cx$$



$$\dot{x} = k\left(1 - \frac{x}{s}\right)x - cx \text{ mit relativer Fangquote } cx$$



$$\dot{x} = k\left(1 - \frac{x}{s}\right)x - cx \text{ mit relativer Fangquote } cx$$



$$\dot{x} = k\left(1 - \frac{x}{s}\right)x - c \text{ mit konstanter Fangquote } c$$

